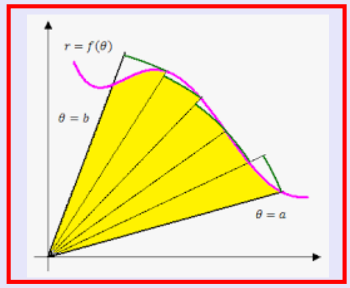
Calculus II

Lecture 11



Feb 19-8:47 AM

1) Find f_{ave} for $f(x) = \dfrac{e^{3x}}{e^{2x}+4}$ over $[0, 1]$ ($a = 0$, $b = 1$)

$f(x) > 0$

$$f_{ave} = \frac{1}{1-0}\int_0^1 \frac{e^{3x}}{e^{2x}+4}\,dx$$

$$= \int_0^1 \frac{e^{2x}\cdot e^{x}}{e^{2x}+2^2}\,dx$$

Let $u = e^x$

$u^2 = (e^x)^2 = e^{2x}$

$du = e^x dx$

$x = 0 \to u = e^0 = 1$

$x = 1 \to u = e^1 = e$

$$= \int_1^e \frac{u^2}{u^2+2^2}\,du$$

$$= \int_1^e \frac{u^2+4-4}{u^2+4}\,du = \int_1^e \left[\frac{u^2+4}{u^2+4} - \frac{4}{u^2+4}\right]du$$

$$= \int_1^e 1\,du - 4\int_1^e \frac{1}{u^2+4}\,du$$

$$= u\Big|_1^e - 4\cdot\frac{1}{2}\tan^{-1}\frac{u}{2}\Big|_1^e =$$

Jun 27-8:01 AM

2) Evaluate $\displaystyle\int \frac{\cos\theta}{\sin^2\theta + 4\sin\theta - 5}\,d\theta$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$= \int \frac{1}{u^2+4u-5}\,du$$

$$\frac{1}{u^2+4u-5} = \frac{1}{(u+5)(u-1)} = \frac{A}{u+5} + \frac{B}{u-1}$$

$$= \frac{A(u-1)+B(u+5)}{(u+5)(u-1)}$$

$$1 = A(u-1) + B(u+5)$$

$u=1 \qquad 1 = A\cdot 0 + B\cdot 6 \rightarrow B = \frac{1}{6}$

$u=-5 \qquad 1 = A\cdot(-6) + B\cdot 0 \rightarrow A = \frac{-1}{6}$

$$\int \frac{1}{u^2+4u-5}\,du = \int \frac{-1/6}{u+5}\,du + \int \frac{1/6}{u-1}\,du$$

$$= -\frac{1}{6}\ln|u+5| + \frac{1}{6}\ln|u-1| + C$$

$$= \frac{1}{6}\Big[\ln|u-1| - \ln|u+5|\Big] + C$$

$$= \frac{1}{6}\ln\left|\frac{u-1}{u+5}\right| + C = \ln\sqrt[6]{\left|\frac{u-1}{u+5}\right|} + C$$

$$= \ln\sqrt[6]{\frac{\sin\theta - 1}{\sin\theta + 5}} + C$$

Jun 27-8:10 AM

2) Evaluate $\displaystyle\int \frac{\cos\theta}{\sin^2\theta + 4\sin\theta - 5}\,d\theta$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$= \int \frac{1}{u^2+4u-5}\,du$$

$$\int \frac{1}{u^2+4u-5}\,du = \int \frac{1}{u^2+4u+4-4-5}\,du$$

$$= \int \frac{1}{(u+2)^2 - 9}\,du$$

Let $w = u+2$

$dw = du$

$$= \int \frac{1}{w^2-3^2}\,dw = \frac{1}{2\cdot 3}\ln\left|\frac{w-3}{w+3}\right| + C$$

using #21

$a = 3$

$$= \frac{1}{6}\ln\left|\frac{u+2-3}{u+2+3}\right| + C$$

$$= \frac{1}{6}\ln\left|\frac{u-1}{u+5}\right| + C$$

$$= \frac{1}{6}\ln\left|\frac{\sin\theta - 1}{\sin\theta + 5}\right| + C$$

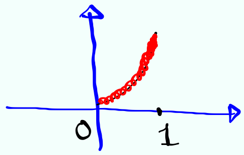
Jun 27-8:10 AM

Find the arc length of $f(x) = x^2$ from $x = 0$ to $x = 1$.

$$L = \int_0^1 \sqrt{1 + [f'(x)]^2}\, dx$$

$f'(x) = 2x$

$[f'(x)]^2 = 4x^2$



$$= \int_0^1 \sqrt{1 + 4x^2}\, dx$$

$u = 2x \qquad u^2 = 4x^2$

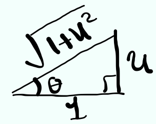
$du = 2\,dx$

$\frac{du}{2} = dx$

$$= \int_0^2 \sqrt{1 + u^2}\, \frac{du}{2}$$

Let $u = 1 \tan\theta$

$\tan\theta = \frac{u}{1}$



$u = \tan\theta$

$1 + u^2 = 1 + \tan^2\theta = \sec^2\theta$

$du = \sec^2\theta\, d\theta$

$\sqrt{a^2 - x^2}$

$\sqrt{x^2 - a^2}$

$\sqrt{x^2 + a^2} \qquad x = a\tan\theta$

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$$= \int_0^2 \sqrt{1 + u^2}\, \frac{du}{2} = \frac{1}{2} \int_0^{\tan^{-1} 2} \sqrt{\sec^2\theta} \cdot \sec^2\theta\, d\theta$$

$u = \tan\theta \qquad u = 0 \qquad \tan\theta = 0 \qquad \theta = 0$

$du = \sec^2\theta\, d\theta \qquad u = 2 \qquad \tan\theta = 2 \qquad \theta = \tan^{-1} 2$

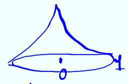
$1 + u^2 = \sec^2\theta$

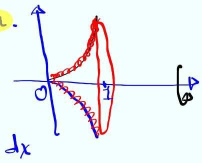
$$= \frac{1}{2} \int \sec^3\theta\, d\theta$$

$$= \frac{1}{2} \left(\text{See Notes} \right) \Big|_0^{\tan^{-1} 2}$$

Jun 27-8:34 AM

Take the arc along $f(x)=x^2$ from $x=0$ to $x=1$, rotate about x-axis.
Find its surface area.





$$S=\int_0^1 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$=\int_0^1 2\pi\cdot x^2\sqrt{1+4x^2}\,dx$$

$u=2x$

$du=2dx$

$u^2=4x^2$

$\frac{u^2}{4}=x^2$

$$=\int_0^2 \cancel{2}\pi\cdot\frac{u^2}{4}\sqrt{1+u^2}\,\frac{du}{\cancel{2}}$$

$$=\frac{\pi}{4}\int_0^2 u^2\sqrt{1+u^2}\,du$$

$u=\tan\theta$

$du=\sec^2\theta\,d\theta$

$u^2=\tan^2\theta$

$1+u^2=\sec^2\theta$

$$=\frac{\pi}{4}\int_0^{\tan^{-1}2}\tan^2\theta\cdot\sqrt{\sec^2\theta}\cdot\sec^2\theta\,d\theta$$

$$=\frac{\pi}{4}\int_0^{\tan^{-1}2}\tan^2\theta\,\sec^3\theta\,d\theta$$

Integration by Parts

$u=\tan^2\theta \quad dv=\sec^3\theta\,d\theta$

$du=2\tan\theta\cdot\sec^2\theta \quad V=\int\sec^3\theta\,d\theta$

$$\int(\sec^2\theta-1)\sec^3\theta\,d\theta$$

$$=\int(\sec^5\theta-\sec^3\theta)\,d\theta$$

Jun 27-8:39 AM

$$\int\sec^n x\,dx=\frac{1}{n-1}\sec^{n-2}x\tan x+\frac{n-2}{n-1}\int\sec^{n-2}x\,dx$$

$n=5$

$$\int\sec^5 x\,dx=\frac{1}{4}\sec^3 x\tan x+\frac{3}{4}\int\sec^3 x\,dx$$

$$\int(\sec^5 x-\sec^3 x)\,dx=\frac{1}{4}\sec^3 x\tan x+\frac{3}{4}\int\sec^3 x\,dx-\int\sec^3 x\,dx$$

$$=\frac{1}{4}\sec^3 x\tan x-\frac{1}{4}\int\sec^3 x\,dx$$

$$=\frac{1}{4}\sec^3 x\tan x-\frac{1}{4}\left[\frac{1}{2}\sec x\tan x+\frac{1}{2}\int\sec x\,dx\right]$$

$$=\frac{1}{4}\sec^3 x\tan x-\frac{1}{8}\sec x\tan x-\frac{1}{8}\int\sec x\,dx$$

$$=\frac{1}{4}\sec^3 x\tan x-\frac{1}{8}\sec x\tan x-\frac{1}{8}\ln|\sec x+\tan x|+C$$

Jun 27-8:52 AM